32.59. **Model:** The magnetic field is that of a current in the wire.
Visualize: Please refer to Figure P32.59.
Solve: As given in Equation 32.6 for a current carrying small segment $\Delta\vec{s}$, the Biot-Savart law is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

For the straight sections, $\Delta\vec{s}\times\hat{r} = 0$ because both $\Delta\vec{s}$ and $\hat{r}$ point along the same line. That is not the case with the curved section over which $\Delta\vec{s}$ and $\vec{r}$ are perpendicular. Thus,

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2} = \frac{\mu_0}{4\pi}\frac{IRd\theta}{R^2} = \frac{\mu_0 Id\theta}{4\pi R}$$

where we used $\Delta s = R\Delta\theta \approx Rd\theta$ for the small arc length Δs. Integrating to obtain the total magnetic field at the center of the semicircle,

$$B = \int_{-\pi/2}^{\pi/2}\frac{\mu_0 Id\theta}{4\pi R} = \frac{\mu_0 I}{4\pi R}\pi = \frac{\mu_0 I}{4R}$$